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PLASTIC DESIGN OF SEISMIC RESISTANT STEEL FRAMES

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SUMMARY

A new method for designing moment resisting steel frames failing in a global mode is presented in this paper. Starting from the analysis of the typical collapse mechanisms of frames subjected to horizontal forces, the method is based on the application of the kinematic theorem of plastic collapse. The beam section properties are assumed to be known quantities, because they are designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections. They are determined by means of design conditions expressing that the kinematically admissible multiplier of the horizontal forces corresponding to the global mechanism has to be the smallest among all kinematically admissible multipliers.

In addition, the proposed design method includes both the influence of distributed loads acting on the beams and the influence of second-order effects. In particular, the influence of second-order effects, which can play an important role in the seismic design of steel frames, is accounted for by the mechanism equilibrium curves of the analysed collapse mechanisms.

Moreover, in order to show the practical application of the proposed design procedure, a worked example is presented. Finally, the inelastic behaviour of the designed frame is compared to that obtained when the simple member hierarchy criterion or a similar rule is applied.

KEY WORDS: plastic design; steel frames; second-order effects; member hierarchy criterion; local ductility

INTRODUCTION

The concept of a desirable member hierarchy in the energy dissipating mechanisms, to be exploited in ductile moment resisting multistorey frames during severe earthquakes, is universally recognized.[1–6] In particular, it is usually required that plastic hinges develop in beams rather than in columns. The aim of this design goal is to avoid collapse mechanisms characterized by poor energy dissipation capacity, such as 'soft-storey' mechanisms.

Due to the predominantly inelastic nature of the structural response during destructive earthquakes, time-history analyses of the inelastic dynamic response of frames subjected to a given ground motion provide the most reliable information on the inelastic performances of structures. Unfortunately, these are analyses rather than design techniques, so that the common design practice still requires the use of engineering judgement in quantifying the member hierarchy to be applied to control the failure mode. For this reason, significant research efforts have been spent in order to establish simple design criteria to avoid undesired collapse mechanisms which undermine the global ductility supply and energy dissipation capacity of structures.

The first studies on this topic have been carried out mainly with reference to reinforced concrete frames[6–9] and, in particular, in New Zealand where the capacity design procedure has been in use since 1980.[7–10] Nowadays, the need to control the location of dissipative zones is recognized to be of primary importance

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independently of the constructional material.[11–15] Therefore, the first step in the seismic design of dissipative structures is the selection of a suitable location of dissipative zones, i.e. a suitable collapse mechanism.

Non-dissipative parts of dissipative seismic resistant structures and their connections to the dissipative ones have to be designed with sufficient overstrength to allow the cyclic yielding of the dissipative parts.

This means that non-dissipative parts have to be designed in order to remain in the elastic range and, therefore, they have to be proportioned on the basis of the maximum internal actions that the dissipative zones are able to transmit. Conversely, the dissipative zones have to be proportioned on the basis of internal actions arising from the seismic forces prescribed by the codes.

This is the general criterion for designing seismic-resistant dissipative structures and represents the so-called 'capacity design approach'. This definition means that non-dissipative parts have to be designed for the 'capacity' of the fully yielded and strain-hardened dissipative zones.[16]

As already stated, regarding the location of dissipative zones, plastic hinges should be located in the beams rather than in the columns. The primary aim of the capacity design of columns is to eliminate the likelihood of the simultaneous formation of plastic hinges at both ends of all the columns of a storey.[4] This design goal is attained by imposing that the sum of the flexural strength of the columns connected to any joint is greater than the sum of the flexural strength of the beams connected to the same joint. With some small differences depending also on the constructional material, this member hierarchy criterion is suggested in all modern codes.[11–14]

With reference to moment resisting steel frames, this simple design criterion can be sufficient to ensure that a 'soft-storey' will not develop, but it does not lead to frames failing in a global mode.[17,18] For this reason, a more sophisticated design procedure, assuring the development of a collapse mechanism of global type, is presented in this paper. The proposed design procedure, based on the kinematic theorem of plastic collapse and on second order plastic analysis, covers an important gap in the design tools for seismic resistant steel frames.

In fact, the theory of limit design has been mainly used in order to compute the collapse multiplier of a given structure.[19] In the case of frames, the most known methods for reaching this purpose are the elementary mechanism combination method (Neal and Symonds method) and the moment distribution method (Horne method). The moment distribution method has been also applied to search structural solutions leading to the minimum weight.[20] However, the plastic methods of structural analysis can be particularly useful to evaluate the inelastic behaviour of seismic resistant frames provided that a suitable distribution of horizontal forces, increasing according to a common multiplier, is selected.[21,22]

The use of second order plastic analysis, i.e. of mechanism equilibrium curves, has been recently suggested as an effective method to include P–Δ effects into the design process of seismic resistant steel frames.[23] The same approach has been also applied within a design procedure devoted to semi-rigid composite frames, even though exact expressions for all collapse mechanisms have not been derived due to the number of variables involved, so that the occurrence of the beam sidesway mechanism, i.e. of the global mechanism, has been investigated through an extensive numerical simulation of the inelastic response of designed frames.[24]

The structural design oriented to the failure mode control is a relatively recent problem which has arisen from seismic design needs, which so far has been mainly treated through simplified rules provided in seismic codes. As already stated, modern seismic codes[11–14] require only the fulfilment of the member hierarchy criterion which is sufficient to avoid storey mechanisms, but it does not allow the complete development of a global mechanism.

Starting from this consideration, a new design method has been proposed[25] aiming at the control of the failure mode of seismic resistant steel frames. It is based on the observation[21,22] that the collapse mechanisms of frames under horizontal forces can be considered as belonging to three main types (Figure 1). The collapse mechanism of the global type is a particular case of type-2 mechanism. The control of the failure mode can be performed through the analysis of $3n_s$ mechanisms (where n_s is the number of storeys).[25]

It is assumed that the beam sections are already designed to resist vertical loads. Therefore, the values of the plastic section modulus of columns only have to be defined so that the kinematically admissible multiplier of the horizontal forces corresponding to the global mechanism is less than those corresponding to the other $3n_s - 1$ kinematically admissible mechanisms. It means that, according to the upper bound theorem,

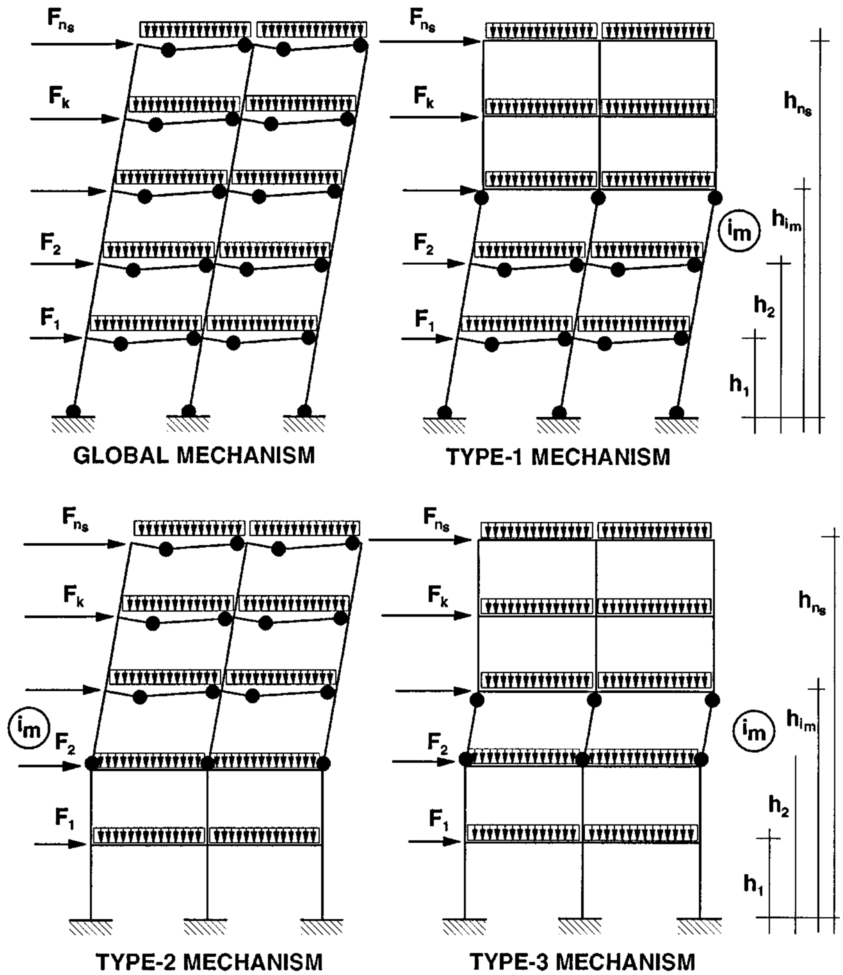

Figure 1. Analysed collapse mechanism typologies

the above stated multiplier is the true collapse multiplier and, therefore, the true collapse mechanism is the global failure mode.

In the original method[25] second-order effects, which are not included in the limit analysis, have been indirectly covered by means of an approximated method based on the use of correction factors. In this paper, from the theoretical point of view, a significant improvement is obtained by accounting for second-order effects by a procedure based on the equilibrium curves of the considered mechanisms, without any significant increase of the computational effort. As it will be emphasized through a worked example aiming to show the practical application of the proposed design procedure, this also allows to account, into the design process, for the plastic rotation capacity of beams and/or beam-to-column connections. In addition, the influence of distributed vertical loads on the location of plastic hinges is analysed.

Static inelastic analyses are successively used to compare the inelastic behaviour of the frame, designed according to the proposed procedure, to that of the same frame designed both according to the simple member hierarchy criterion adopted by Eurocode 8[11] and according to a similar criterion recently proposed.[18]

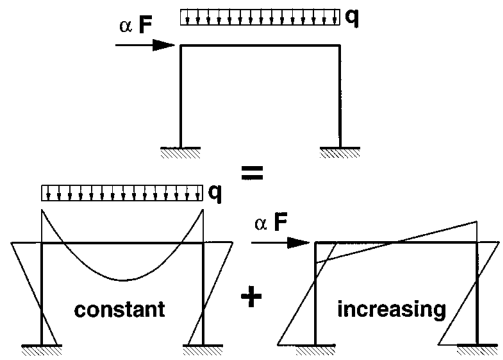

Figure 2. Superposition of vertical loads and horizontal forces

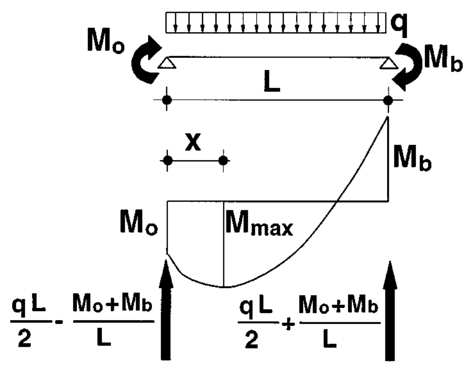

Figure 3. Location of the second plastic hinge

LOCATION OF BEAM PLASTIC HINGES

Concerning the location of the plastic hinges in the beams, it is important to stress that it depends on the magnitude of the uniform load acting on the beams.

The seismic action is modelled through a system of horizontal forces whose distribution can be selected according to a proper combination of the eigenmodes. The magnitude of these horizontal forces is governed by the multiplier α, while the vertical loads are assumed to be constant. For this reason, at any loading stage characterized by a given value of the horizontal force multiplier α, the bending moment diagram of the beams is the superposition of that due to vertical loads and that due to horizontal forces. It means that, increasing the horizontal forces (i.e. the multiplier α), the first plastic hinge is always developed at the beam end opposite to the horizontal forces (Figure 2).

Regarding the location of the second plastic hinge in a beam, it is strictly dependent on the magnitude of the vertical loads. This location can be determined by taking into account that the beam plastic moment M_{b} acts at one end, where the first plastic hinge is formed, while at the second end there is a bending moment M_0 which progressively increases due to the progressive increase of the horizontal forces (Figure 3). The maximum bending moment is attained at the abscissa given by

$$x = \frac{L}{2} - \frac{M_0 + M_{\mathrm{b}}}{qL} \tag{1}$$

where L is the beam length and q is the uniform load acting on the beam.

It is easy to recognize that the second plastic hinge forms at the first end provided that $M_0 = M_\mathrm{b}$ (yielding condition) and $x \leqslant 0$. By imposing these conditions, it is derived that plastic hinges form at both beam ends provided that the uniform load satisfies the following limitation:

$$q \leqslant q_{\mathrm{lim}} = \frac{4M_\mathrm{b}}{L^2} \tag{2}$$

Now, it is interesting to compute the abscissa where the second plastic hinge forms when the uniform load exceeds the limit value provided by equation (2).

The maximum bending moment, which occurs at the abscissa provided by equation (1), is given by

$$M_{\mathrm{max}} = \frac{M_0 - M_\mathrm{b}}{2} + \frac{qL^2}{8} + \frac{(M_0 + M_\mathrm{b})^2}{2qL^2} \tag{3}$$

By imposing the yielding condition $M_{\mathrm{max}} = M_\mathrm{b}$, relationship (3) provides a second-order equation whose positive solution is given by

$$M_0 = 2(M_\mathrm{b} q L^2)^{1/2} - M_\mathrm{b} - \frac{qL^2}{2} \tag{4}$$

which represents the value of the end moment M_0 corresponding to the occurrence of the second plastic hinge at the abscissa provided by equation (1).

By combining equation (4) with equation (1), the abscissa where the second plastic hinge forms (when the uniform load exceeds $4M_\mathrm{b}/L^2$) is given by

$$x = L - 2\left(\frac{M_\mathrm{b}}{q}\right)^{1/2} \tag{5}$$

EQUILIBRIUM CURVES OF ANALYSED MECHANISMS

Notation

n_s	number of storeys
n_c	number of columns
n_b	number of bays
k	storey index
i	column index
j	bay index
i_m	mechanism index
L_j	span of the jth bay
$M_{\mathrm{c},ik}$	plastic moment, reduced for the presence of the axial internal force, of the ith column of the kth storey
$M_{\mathrm{b},jk}$	plastic moment of the jth beam of the kth storey
q_{jk}	uniform vertical load acting on the jth beam of the kth storey
x_{jk}	abscissa of the second plastic hinge of the jth beam of the kth storey, given by $$x_{jk} = L_j - 2\left(\frac{M_{\mathrm{b},jk}}{q_{jk}}\right)^{1/2} \quad \text{for } q_{jk} > \frac{4M_{\mathrm{b},jk}}{L_j^2} \tag{6}$$ while $x_{jk} = 0$ in the opposite case
$R_{\mathrm{b},jk}$	coefficient related to the participation of the jth beam of the kth storey to the collapse mechanism; in addition this coefficient accounts for the magnitude of the rotations of the plastic hinges (Figure 4) giving $$R_{\mathrm{b},jk} = \frac{L_j}{L_j - x_{jk}} \tag{7}$$

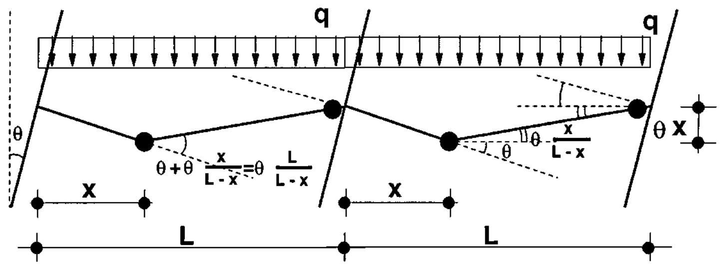

Figure 4. Rotation of beam plastic hinges

when the jth beam of the kth storey participates in the collapse mechanism and $R_{b,jk} = 0$ in the opposite case

$R_{c,ik}$ coefficient accounting for the participation of the ith column of the kth storey to the collapse mechanism. $R_{c,ik} = 2$ when the column is yielded at both ends, $R_{c,ik} = 1$ when only one column end is yielded, and, $R_{c,ik} = 0$ when the column does not participate in the collapse mechanism

$D_{v,jk}$ coefficient, related to the external work of the uniform load acting on the jth beam of the kth storey (Figure 4), given by

$$D_{v,jk} = \frac{L_j x_{jk}}{2} \tag{8}$$

when the jth beam of the kth storey participates in the collapse mechanism and $D_{v,jk} = 0$ in the opposite case

$\mathbf{F}^T = \{F_1, F_2, \dots, F_k, \dots, F_{n_s}\}$ is the vector of the design horizontal forces, where F_k is the horizontal force applied to the kth storey

$\mathbf{h}^T = \{h_1, h_2, \dots, h_k, \dots, h_{n_s}\}$ is the vector of storey heights, where h_k is the height of the kth storey

$\mathbf{s}$ is the shape vector of the storey horizontal virtual displacements ($\mathrm{d}\mathbf{u} = \mathbf{s}\,\mathrm{d}\theta$, where $\mathrm{d}\theta$ is the virtual rotation of the plastic hinges of the columns involved in the mechanism);

$\mathbf{V}^T = \{V_1, V_2, \dots, V_k, \dots, V_{n_s}\}$ is the vector of the storey vertical loads, where V_k is the total load acting at the kth storey given by

$$V_k = \sum_{j=1}^{n_b} q_{jk} L_j \tag{9}$$

$\mathbf{B}$ matrix of order $n_b \times n_s$ whose elements B_{jk} are equal to the plastic moments of beams (i.e. $B_{jk} = M_{b,jk}$)

$\mathbf{C}$ matrix of order $n_c \times n_s$ whose elements C_{ik} are equal to the column plastic moments (i.e. $C_{ik} = M_{c,ik}$);

$\mathbf{R}_b$ matrix (order $n_b \times n_s$) of the coefficients $R_{b,jk}$

$\mathbf{R}_c$ matrix (order $n_c \times n_s$) of the coefficients $R_{c,ik}$

$\mathbf{D}_v$ matrix (order $n_b \times n_s$) of the coefficients $D_{v,jk}$

$\mathbf{M}_{ck}^T = \{M_{c,1k}, M_{c,2k}, \dots, M_{c,ik}, \dots, M_{c,n_c k}\}$ is the vector of the plastic moments of the columns of the kth storey, reduced due to the influence of the axial force

$\mathbf{q}$ matrix (order $n_b \times n_s$) of the uniform loads acting on the beams

Mechanism equilibrium curves

As already pointed out, the collapse mechanisms of moment resisting frames under seismic horizontal forces can be considered as belonging to three main typologies (Figure 1). The collapse mechanism of the global type is a particular case of type 2 mechanism.

The linearized mechanism equilibrium curve can be always expressed as

$$\alpha_c = \alpha - \gamma\delta \tag{10}$$

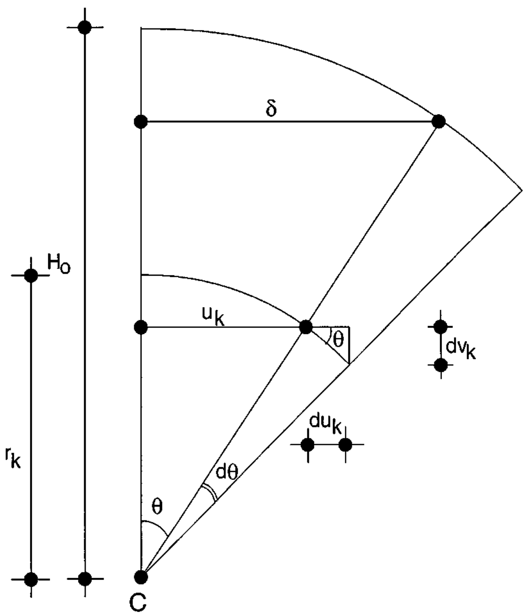

Figure 5. Second order vertical displacements

where α is the kinematically admissible multiplier of horizontal forces and γ is the slope of the mechanism equilibrium curve.

Concerning the evaluation of the kinematically admissible multiplier of horizontal forces corresponding to the generic mechanism, it is easy to recognize that, for a virtual rotation $\mathrm{d}\theta$ of the plastic hinges of the columns involved in the mechanism, the internal work can be expressed as

$$W_{\mathrm{i}} = [\mathrm{tr}(\mathbf{C}^{\mathrm{T}}\mathbf{R}_{\mathrm{c}}) + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}_{\mathrm{b}})]\,\mathrm{d}\theta \tag{11}$$

where tr denotes the trace of the matrix.

The external work due to the horizontal forces and to the uniform load acting on the beams can be written as

$$W_{\mathrm{e}} = [\alpha\mathbf{F}^{\mathrm{T}}\mathbf{s} + \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}})]\,\mathrm{d}\theta \tag{12}$$

Therefore the application of the virtual work principle provides the kinematically admissible multiplier as

$$\alpha = \frac{[\mathrm{tr}(\mathbf{C}^{\mathrm{T}}\mathbf{R}_{\mathrm{c}}) + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}_{\mathrm{b}}) - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}})]}{\mathbf{F}^{\mathrm{T}}\mathbf{s}} \tag{13}$$

In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second-order work due to vertical loads. With reference to Figure 5, it can be observed that the horizontal displacement of the kth storey involved in the generic mechanism is given by $u_k = r_k \sin\theta$, where r_k is the distance of the kth storey from the centre of rotation C and θ the angle of rotation.

The top sway displacement is given by $\delta = H_0 \sin\theta$, where H_0 is the sum of the interstorey heights of the storeys involved by the generic mechanism.

The relationship between vertical and horizontal virtual displacements is given by $\mathrm{d}v_k = \mathrm{d}u_k \tan\theta \approx \mathrm{d}u_k \sin\theta$. It shows that, as the ratio $\mathrm{d}v_k/\mathrm{d}u_k$ is independent of the considered storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by $\mathrm{d}u_k = r_k \cos\theta\,\mathrm{d}\theta \approx r_k\,\mathrm{d}\theta$, where r_k defines the shape of the virtual horizontal displacement vector, while the virtual vertical displacements are given by $\mathrm{d}v_k = (\delta/H_0)\,r_k\,\mathrm{d}\theta$ and, therefore, they have the same

shape r_k of the horizontal ones. It can be concluded that

$$\mathrm{d}\mathbf{v} = \frac{\delta}{H_0}\mathbf{s}\,\mathrm{d}\theta \tag{14}$$

As a consequence, the second-order work due to vertical loads is given by

$$W_{\mathrm{v}} = \mathbf{V}^{\mathrm{T}}\mathbf{s}\frac{\delta}{H_0}\mathrm{d}\theta \tag{15}$$

Therefore, the slope of the mechanism equilibrium curve is given by

$$\gamma = \frac{\mathbf{V}^{\mathrm{T}}\mathbf{s}\dfrac{1}{H_0}}{\mathbf{F}^{\mathrm{T}}\mathbf{s}} \tag{16}$$

The following notation will be used to denote the parameters of the equilibrium curve of the considered mechanisms:

$\alpha^{(\mathrm{g})}$ and $\gamma^{(\mathrm{g})}$ are, kinematically admissible multiplier of the horizontal forces (rigid–plastic theory) and the slope of the softening branch of the α–δ curve, corresponding to the global-type mechanism

$\alpha_{i_{\mathrm{m}}}^{(t)}$ and $\gamma_{i_{\mathrm{m}}}^{(t)}$ have the same meaning of the previous symbols, but they are referred to the i_{m}th mechanism of the tth type ($t = 1, 2, 3$)

The expressions of the above parameters will be furtherly developed in order to evidence the contribution of the columns to the internal work.

Global-type mechanism

In the case of global-type mechanism (Figure 1), the shape vector of the horizontal displacements is given by $\mathbf{s}^{(\mathrm{g})} = \mathbf{h}$. In addition, as all storeys participate to the collapse mechanism, all beams are involved. This is taken into account through the matrix $\mathbf{R}_{\mathrm{b}}^{(\mathrm{g})}$ related to the rotation of the plastic hinges and the matrix $\mathbf{D}_{\mathrm{v}}^{(\mathrm{g})}$ related to the beam vertical displacements. $\mathbf{R}_{\mathrm{b}}^{(\mathrm{g})}$ is the value of $\mathbf{R}_{\mathrm{b}}$ and $\mathbf{D}_{\mathrm{v}}^{(\mathrm{g})}$ is the value of $\mathbf{D}_{\mathrm{v}}$ for the specific case of global mechanism.

Therefore, the kinematically admissible multiplier is given by

$$\alpha^{(\mathrm{g})} = \frac{\mathbf{M}_{\mathrm{c}1}^{\mathrm{T}}\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}_{\mathrm{b}}^{(\mathrm{g})}) - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}}^{(\mathrm{g})})}{\mathbf{F}^{\mathrm{T}}\mathbf{s}^{(\mathrm{g})}} \tag{17}$$

where $\mathbf{I}$ is the unit vector of order n_{c}. In addition, taking into account that $H_0 = h_{n_{\mathrm{s}}}$, because all storeys are involved in the collapse mechanism, the slope $\gamma^{(\mathrm{g})}$ of the mechanism equilibrium curve is obtained from equation (16) for $\mathbf{s} = \mathbf{s}^{(\mathrm{g})}$ and $H_0 = h_{n_{\mathrm{s}}}$.

Type-1 mechanisms

With reference to the i_{m}th mechanism of type-1 (Figure 1), the shape vector of the horizontal displacements can be written as

$$\mathbf{s}_{i_{\mathrm{m}}}^{(1)\mathrm{T}} = \{h_1, h_2, h_3, \ldots, h_{i_{\mathrm{m}}}, h_{i_{\mathrm{m}}}, h_{i_{\mathrm{m}}}\} \tag{18}$$

where the first element equal to $h_{i_{\mathrm{m}}}$ corresponds to the i_{m}th component. The kinematically admissible multiplier corresponding to the i_{m}th mechanism of type-1 is given by

$$\alpha_{i_{\mathrm{m}}}^{(1)} = \frac{\mathbf{M}_{\mathrm{c}1}^{\mathrm{T}}\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}_{\mathrm{b}_{i_{\mathrm{m}}}}^{(1)}) + \mathbf{M}_{\mathrm{c}_{i_{\mathrm{m}}}}^{\mathrm{T}}\mathbf{I} - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}_{\mathrm{v}_{i_{\mathrm{m}}}}^{(1)})}{\mathbf{F}^{\mathrm{T}}\mathbf{s}_{i_{\mathrm{m}}}^{(1)}} \tag{19}$$

where $\mathbf{R}_{\mathrm{b}_{i_{\mathrm{m}}}}^{(1)}$ is the value of $\mathbf{R}_{\mathrm{b}}$ for the i_{m}th mechanism of this type and $\mathbf{D}_{\mathrm{v}_{i_{\mathrm{m}}}}^{(1)}$ is the value of $\mathbf{D}_{\mathrm{v}}$ for the i_{m}th mechanism of type-1. In addition, only the first i_{m} storeys participate in the collapse mechanism, so that

$H_0 = h_{i_m}$. As a consequence, the slope $\gamma_{i_m}^{(1)}$ of the mechanism equilibrium curve is still computed through equation (16), but assuming $\mathbf{s} = \mathbf{s}_{i_m}^{(1)}$ and $H_0 = h_{i_m}$.

Type-2 mechanisms

With reference of the i_mth mechanism of type-2 (Figure 1), the shape vector of the horizontal displacements can be written as

$$\mathbf{s}_{i_m}^{(2)\mathrm{T}} = \{0, 0, 0, 0, h_{i_m} - h_{i_m-1}, h_{i_m+1} - h_{i_m-1}, \ldots, h_{n_s} - h_{i_m-1}\} \tag{20}$$

where the first non-zero element is the i_mth one.

The kinematically admissible multiplier corresponding to the i_mth mechanism of type-2 is given by

$$\alpha_{i_m}^{(2)} = \frac{\mathbf{M}_{c_{i_m}}^{\mathrm{T}}\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}_{b_{i_m}}^{(2)}) - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}_{v_{i_m}}^{(2)})}{\mathbf{F}^{\mathrm{T}}\mathbf{s}_{i_m}^{(2)}} \tag{21}$$

where $\mathbf{R}_{b_{i_m}}^{(2)}$ is the value of $\mathbf{R}_b$ for the i_mth mechanism of type-2 and $\mathbf{D}_{v_{i_m}}^{(2)}$ is the corresponding value of the matrix $\mathbf{D}_v$.

In addition, the i_mth storey and those above it participate in the mechanism. Therefore, the slope of the mechanism equilibrium curve is obtained from equation (16) with $H_0 = h_{n_s} - h_{i_m-1}$ and $\mathbf{s} = \mathbf{s}_{i_m}^{(2)}$.

Type-3 mechanisms

Finally, with reference to the i_mth mechanism of type-3 (Figure 1), the shape vector of the horizontal displacements can be written as:

$$\mathbf{s}_{i_m}^{(3)\mathrm{T}} = \{0, 0, \ldots, 0, 1, 1, 1, \ldots, 1\}\,(h_{i_m} - h_{i_m-1}) \tag{22}$$

where the first term different from zero is the i_mth one.

Moreover, both the matrix $\mathbf{R}_{b_{i_m}}^{(3)}$ and the matrix $\mathbf{D}_{v_{i_m}}^{(3)}$ are null matrices, because in this case there is not any beam participating to the collapse mechanism. Therefore, the kinematically admissible multiplier of the i_mth mechanism of type-3 is given by

$$\alpha_{i_m}^{(3)} = \frac{2\mathbf{M}_{c_{i_m}}^{\mathrm{T}}\mathbf{I}}{\mathbf{F}^{\mathrm{T}}\mathbf{s}_{i_m}^{(3)}} \tag{23}$$

which accounts for the fact that the columns of the i_mth storey are yielded at both ends.

As the i_mth storey only is involved in the mechanism $H_0 = h_{i_m} - h_{i_m-1}$, and the corresponding slope $\gamma_{i_m}^{(3)}$ of the mechanism equilibrium curve can be obtained by substituting this value in equation (16) where also $\mathbf{s} = \mathbf{s}_{i_m}^{(3)}$ has to be assumed.

FAILURE MODE CONTROL

Design conditions

In order to design frames failing in a global mode, the cross-sections of columns have to be dimensioned so that, according to the upper bound theorem, the kinematically admissible horizontal force multiplier corresponding to the global-type mechanism is the minimum among all kinematically admissible multipliers.

This condition is sufficient to assure the desired collapse mechanism provided that the structural material behaves as rigid–plastic so that the horizontal displacements are equal to zero up to the complete development of the collapse mechanism. On the contrary, the actual behaviour is elastoplastic with significant displacements before the complete development of the collapse mechanism. These displacements give rise to second order effects which cannot be neglected in the design process.

From the practical point of view, the influence of second order effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms. However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement δ_u which has to be compatible with the plastic rotation capacity of members (Figure 6).

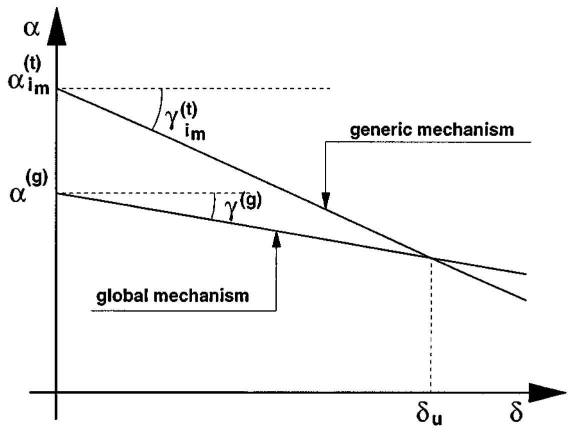

Figure 6. Design conditions

Therefore, the following design conditions have to be imposed:

$$\alpha^{(g)} - \gamma^{(g)}\,\delta_u \leqslant \alpha^{(t)}_{i_m} - \gamma^{(t)}_{i_m}\,\delta_u, \quad i_m = 1, 2, 3, \ldots, n_s, \quad t = 1, 2, 3 \tag{24}$$

This means that there are $3n_s$ design conditions to be satisfied in the case of a frame having n_s storeys. These conditions, which derive directly from the extension of the upper bound theorem to the mechanism equilibrium curves, will be integrated by conditions related to technological limitations.

Conditions to avoid type-1 mechanisms

The n_s-conditions expressed by relationship (24) for $t = 1$ provide:

$$\frac{\mathbf{M}^{\mathrm{T}}_{c1}\,\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}^{(g)}_{b}) - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}^{(g)}_{v})}{\mathbf{F}^{\mathrm{T}}\,\mathbf{s}^{(g)}} - \frac{\mathbf{V}^{\mathrm{T}}\,\mathbf{s}^{(g)}\,\dfrac{\delta_u}{h_{n_s}}}{\mathbf{F}^{\mathrm{T}}\,\mathbf{s}^{(g)}}$$

$$\leqslant \frac{\mathbf{M}^{\mathrm{T}}_{c1}\,\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}^{(1)}_{b_{i_m}}) + \mathbf{M}^{\mathrm{T}}_{c_{i_m}}\mathbf{I} - \mathrm{tr}(\mathbf{q}^{\mathrm{T}}\mathbf{D}^{(1)}_{v_{i_m}})}{\mathbf{F}^{\mathrm{T}}\,\mathbf{s}^{(1)}_{i_m}} - \frac{\mathbf{V}^{\mathrm{T}}\mathbf{s}^{(1)}_{i_m}\,\dfrac{\delta_u}{h_{i_m}}}{\mathbf{F}^{\mathrm{T}}\,\mathbf{s}^{(1)}_{i_m}} \tag{25}$$

In order to express the above design conditions in a convenient form, it is useful to introduce the following parameters:

$$\mu^{(g)} = 2\,\mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{R}^{(g)}_{b}) = \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} 2\,M_{b,jk}\,\frac{L_j}{L_j - x_{jk}} \tag{26}$$

and

$$\nu^{(g)} = \frac{1}{h_{n_s}}\mathbf{V}^{\mathrm{T}}\,\mathbf{s}^{(g)} = \frac{1}{h_{n_s}}\sum_{k=1}^{n_s} V_k\,h_k \tag{27}$$

$$\tau^{(g)} = \mathrm{tr}\,(\mathbf{q}^{\mathrm{T}}\mathbf{D}^{(g)}_{v}) = \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} q_{jk}\,\frac{L_j\,x_{jk}}{2} \tag{28}$$

With reference to the global mechanism, the parameter $\mu^{(g)}$ represents the internal work developed by beams, the parameter $\nu^{(g)}$ is related to the second-order work due to vertical loads while the parameter $\tau^{(g)}$ represents the external work due to the uniform vertical loads acting on the beams. All these parameters are known quantities, because both the beam plastic moments and the magnitude of vertical loads are data of the design problem.

In addition, it is useful to introduce the following non-dimensional functions of the mechanism index i_m:

$$\xi_{i_m} = \frac{2\,\mathrm{tr}(\mathbf{B}^T \mathbf{R}^{(1)}_{b_{i_m}})}{2\,\mathrm{tr}(\mathbf{B}^T \mathbf{R}^{(g)}_{b})} = \frac{2\mathrm{tr}(\mathbf{B}^T \mathbf{R}^{(1)}_{b_{i_m}})}{\mu^{(g)}} = \frac{\sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} 2M_{b,jk}\dfrac{L_j}{L_j - x_{jk}}}{\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} 2M_{b,jk}\dfrac{L_j}{L_j - x_{jk}}} \tag{29}$$

$$\lambda_{i_m} = \frac{\mathbf{F}^T \mathbf{s}^{(1)}_{i_m}}{\mathbf{F}^T \mathbf{s}^{(g)}} = \frac{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k}{\sum_{k=1}^{n_s} F_k h_k} \tag{30}$$

$$\zeta^{(1)}_{i_m} = \frac{\mathrm{tr}(\mathbf{q}^T \mathbf{D}^{(1)}_{v_{i_m}})}{\mathrm{tr}(\mathbf{q}^T \mathbf{D}^{(g)}_{v})} = \frac{\mathrm{tr}(\mathbf{q}^T \mathbf{D}^{(1)}_{v_{i_m}})}{\tau^{(g)}} = \frac{\sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} q_{jk}\dfrac{L_j x_{jk}}{2}}{\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} q_{jk}\dfrac{L_j x_{jk}}{2}} \tag{31}$$

The function ξ_{i_m} represents the ratio between the internal work which the beams develop in the i_mth mechanism of type-1 and that developed in the global mechanism. The function λ_{i_m} represents the ratio between the external work which the horizontal forces develop in the i_mth mechanism of type-1 and that developed in the global mechanism. Finally, the function $\zeta^{(1)}_{i_m}$ represents the ratio between the external work which the uniform vertical loads develop in the i_mth mechanism of type-1 and that developed in the global mechanism. The function ξ_{i_m} is known, because the plastic moments of beams $M_{b,jk}$ are known. In fact, the beam sections are designed to resist vertical loads. In addition, both the horizontal forces F_k and the uniform loads q_{jk} are assigned so that λ_{i_m} and $\zeta^{(1)}_{i_m}$ are also known.

Moreover, in order to account for the influence of second-order effects, an additional function related to the slopes of the mechanism equilibrium curves has to be defined:

$$\Delta^{(1)}_{i_m} = \frac{\mathbf{F}^T \mathbf{s}^{(g)}}{\mathbf{F}^T \mathbf{s}^{(1)}_{i_m}}\,\frac{\dfrac{1}{h_{i_m}}\mathbf{V}^T \mathbf{s}^{(1)}_{i_m}}{\dfrac{1}{h_{n_s}}\mathbf{V}^T \mathbf{s}^{(g)}} = \frac{1}{\lambda_{i_m}}\,\frac{\dfrac{1}{h_{i_m}}\mathbf{V}^T \mathbf{s}^{(1)}_{i_m}}{\nu^{(g)}} = \frac{\dfrac{1}{\lambda_{i_m}}\dfrac{1}{h_{i_m}}\left(\sum_{k=1}^{i_m} V_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} V_k\right)}{\dfrac{1}{h_{n_s}}\sum_{k=1}^{n_s} V_k h_k} \tag{32}$$

The parameter $\Delta^{(1)}_{i_m}$ represents the ratio between the slope of the equilibrium curve of the i_mth mechanism of type-1 and that of the global mechanism.

By means of the above parameters and functions, the n_s design conditions to be satisfied in order to avoid collapse mechanisms of type-1 can be expressed as follows:

$$\mathbf{M}^T_{c1}\mathbf{I}\left(1 - \frac{1}{\lambda_{i_m}}\right) + \mu^{(g)}\left(1 - \frac{\xi_{i_m}}{\lambda_{i_m}}\right) + \nu^{(g)}\delta_u(\Delta^{(1)}_{i_m} - 1) + \tau^{(g)}\left(\frac{\zeta^{(1)}_{i_m}}{\lambda_{i_m}} - 1\right) \leqslant \frac{1}{\lambda_{i_m}}\mathbf{M}^T_{c,i_m}\mathbf{I} \tag{33}$$

In addition, it is convenient to introduce the following parameter:

$$\rho_{i_m} = \frac{\mathbf{M}^T_{c,i_m}\mathbf{I}}{\mathbf{M}^T_{c1}\mathbf{I}} = \frac{\sum_{i=1}^{n_c} M_{c,ii_m}}{\sum_{i=1}^{n_c} M_{c,i1}} \tag{34}$$

which is the ratio of the sum of the reduced plastic moments of the i_mth storey columns and the same sum corresponding to the first storey columns. All design conditions will be expressed by means of these ratios.

Denoting by $\rho^{(1)}_{i_m}$ the values of the above ratios which have to be assured to prevent failure according to type-1 collapse mechanisms, the i_mth condition to be satisfied to avoid these collapse mechanisms can be

written in the following form:

$$\rho_{i_m}^{(1)} \geqslant \frac{\left(1-\dfrac{1}{\lambda_{i_m}}\right)\sum_{i=1}^{n_c} M_{c,i1} + \left(1-\dfrac{\xi_{i_m}}{\lambda_{i_m}}\right)\mu^{(g)} + \nu^{(g)}(\Delta_{i_m}^{(1)}-1)\,\delta_u + \tau^{(g)}\left(\dfrac{\zeta_{i_m}^{(1)}}{\lambda_{i_m}}-1\right)}{\dfrac{1}{\lambda_{i_m}}\sum_{i=1}^{n_c} M_{c,i1}} \tag{35}$$

which has to be applied for $i_m = 1, 2, 3, \ldots, n_s$.

Conditions to avoid type-2 mechanisms

The design conditions to be satisfied in order to avoid collapse mechanisms of type-2 can be expressed according to the following relationship:

$$\frac{\mathbf{M}_{c1}^{T}\,\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{T}\mathbf{R}_{b}^{(g)}) - \mathrm{tr}(\mathbf{q}^{T}\mathbf{D}_{v}^{(g)})}{\mathbf{F}^{T}\mathbf{s}^{(g)}} - \frac{\mathbf{V}^{T}\mathbf{s}^{(g)}\dfrac{\delta_u}{h_{n_s}}}{\mathbf{F}^{T}\mathbf{s}^{(g)}}$$

$$\leqslant \frac{\mathbf{M}_{c_{i_m}}^{T}\,\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{T}\mathbf{R}_{b_{i_m}}^{(2)}) - \mathrm{tr}(\mathbf{q}^{T}\mathbf{D}_{v_{i_m}}^{(2)})}{\mathbf{F}^{T}\mathbf{s}_{i_m}^{(2)}} - \frac{\mathbf{V}^{T}\mathbf{s}_{i_m}^{(2)}\dfrac{\delta_u}{h_{n_s}-h_{i_m-1}}}{\mathbf{F}^{T}\mathbf{s}_{i_m}^{(2)}} \tag{36}$$

These n_s conditions can be conveniently rearranged by introducing a new series of parameters:

$$\theta_{i_m} = \frac{2\,\mathrm{tr}(\mathbf{B}^{T}\mathbf{R}_{b_{i_m}}^{(2)})}{2\,\mathrm{tr}(\mathbf{B}^{T}\mathbf{R}_{b}^{(g)})} = \frac{2\,\mathrm{tr}(\mathbf{B}^{T}\mathbf{R}_{b_{i_m}}^{(2)})}{\mu^{(g)}} = \frac{\sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} 2M_{b,jk}\dfrac{L_j}{L_j - x_{jk}}}{\mu^{(g)}} \tag{37}$$

$$\gamma_{i_m} = \frac{\mathbf{F}^{T}\mathbf{s}_{i_m}^{(2)}}{\mathbf{F}^{T}\mathbf{s}^{(g)}} = \frac{\sum_{k=i_m}^{n_s} F_k(h_k - h_{i_m-1})}{\sum_{k=1}^{n_s} F_k h_k} \tag{38}$$

$$\zeta_{i_m}^{(2)} = \frac{\mathrm{tr}(\mathbf{q}^{T}\mathbf{D}_{v_{i_m}}^{(2)})}{\mathrm{tr}(\mathbf{q}^{T}\mathbf{D}_{v}^{(g)})} = \frac{\mathrm{tr}(\mathbf{q}^{T}\mathbf{D}_{v_{i_m}}^{(2)})}{\tau^{(g)}} = \frac{\sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} q_{jk}\dfrac{L_j x_{jk}}{2}}{\sum_{k=1}^{n_s}\sum_{j=1}^{n_b} q_{jk}\dfrac{L_j x_{jk}}{2}} \tag{39}$$

where θ_{i_m}, γ_{i_m} and $\zeta_{i_m}^{(2)}$ are still known functions of the mechanism index i_m, because the plastic moments of beams, the horizontal forces and the uniform vertical loads are input data of the design problem. In addition, it is useful to note that function θ_{i_m} represents the ratio between the internal work done by the beams in the i_mth mechanism of type-2 and that done in the global mechanism. Moreover, the function γ_{i_m} represents the ratio between the external work due to the horizontal forces in the i_mth mechanism of type-2 and that in the global mechanism. Finally, $\zeta_{i_m}^{(2)}$ has the same meaning, but with reference to the external work due to the uniform vertical loads.

Furthermore, additional parameters related to the influence of second-order effects are necessary:

$$\Delta_{i_m}^{(2)} = \frac{\mathbf{F}^{T}\mathbf{s}^{(g)}}{\mathbf{F}^{T}\mathbf{s}_{i_m}^{(2)}}\,\frac{\dfrac{1}{h_{n_s}-h_{i_m-1}}\mathbf{V}^{T}\mathbf{s}_{i_m}^{(2)}}{\dfrac{1}{h_{n_s}}\mathbf{V}^{T}\mathbf{s}^{(g)}} = \frac{1}{\gamma_{i_m}}\,\frac{\dfrac{1}{h_{n_s}-h_{i_m-1}}\mathbf{V}^{T}\mathbf{s}_{i_m}^{(2)}}{\nu^{(g)}}$$

$$= \frac{\dfrac{1}{\gamma_{i_m}}\dfrac{1}{h_{n_s}-h_{i_m-1}}\sum_{k=i_m}^{n_s} V_k(h_k - h_{i_m-1})}{\dfrac{1}{h_{n_s}}\sum_{k=1}^{n_s} V_k h_k} \tag{40}$$

$\Delta_{i_m}^{(2)}$ represents the ratio between the slope of the equilibrium curve of the i_mth mechanism of type-2 and that of the global mechanism.

By means of the above parameters and functions, the n_s design conditions to be satisfied to avoid collapse mechanisms of type-2 can be expressed as follows:

$$\mathbf{M}_{c1}^{T}\,\mathbf{I} + \mu^{(g)}\left(1 - \frac{\theta_{i_m}}{\gamma_{i_m}}\right) + \nu^{(g)}\,\delta_u\,(\Delta_{i_m}^{(2)} - 1) + \tau^{(g)}\left(\frac{\zeta_{i_m}^{(2)}}{\gamma_{i_m}} - 1\right) \leqslant \frac{1}{\gamma_{i_m}}\,\mathbf{M}_{c,i_m}^{T}\,\mathbf{I} \tag{41}$$

Denoting by $\rho_{i_m}^{(2)}$ the values of the ratios (34) corresponding to the fulfilment of the n_s design conditions to avoid collapse mechanisms of type-2, the following relationship is obtained:

$$\rho_{i_m}^{(2)} \geqslant \frac{\sum_{i=1}^{n_c} M_{c,i1} + \mu^{(g)}\left(1 - \dfrac{\theta_{i_m}}{\gamma_{i_m}}\right) + \tau^{(g)}\left(\dfrac{\zeta_{i_m}^{(2)}}{\gamma_{i_m}} - 1\right) + \nu^{(g)}(\Delta_{i_m}^{(2)} - 1)\,\delta_u}{\dfrac{1}{\gamma_{i_m}}\sum_{i=1}^{n_c} M_{c,i1}} \tag{42}$$

which has to be applied for $i_m = 1, 2, 3, \ldots, n_s$.

Conditions to avoid type-3 mechanisms

The design conditions to be satisfied in order to avoid collapse mechanisms of type-3 can be written as

$$\frac{\mathbf{M}_{c1}^{T}\,\mathbf{I} + 2\,\mathrm{tr}(\mathbf{B}^{T}\,\mathbf{R}_{b}^{(g)}) - \mathrm{tr}(\mathbf{q}^{T}\,\mathbf{D}_{v}^{(g)})}{\mathbf{F}^{T}\,\mathbf{s}^{(g)}} - \frac{\mathbf{V}^{T}\,\mathbf{s}^{(g)}\,\dfrac{\delta_u}{h_{n_s}}}{\mathbf{F}^{T}\,\mathbf{s}^{(g)}} \leqslant \frac{2\mathbf{M}_{ci_m}^{T}\,\mathbf{I}}{\mathbf{F}^{T}\,\mathbf{s}_{i_m}^{(3)}} - \frac{\mathbf{V}^{T}\,\mathbf{s}_{i_m}^{(3)}\,\dfrac{\delta_u}{h_{i_m} - h_{i_m-1}}}{\mathbf{F}^{T}\,\mathbf{s}_{i_m}^{(3)}} \tag{43}$$

It is convenient to introduce a new known discrete function of the mechanism index i_m:

$$\beta_{i_m} = \frac{\mathbf{F}^{T}\,\mathbf{s}_{i_m}^{(3)}}{\mathbf{F}^{T}\,\mathbf{s}^{(g)}} = \frac{(h_{i_m} - h_{i_m-1})\sum_{k=i_m}^{n_s} F_k}{\sum_{k=1}^{n_s} F_k\,h_k} \tag{44}$$

which represents the ratio between the external work done by the horizontal forces in the i_mth mechanism of type-3 and that done in the global mechanism.

In addition, the influence of second-order effects can be taken into account by means of the following parameters:

$$\Delta_{i_m}^{(3)} = \frac{\mathbf{F}^{T}\,\mathbf{s}^{(g)}}{\mathbf{F}^{T}\,\mathbf{s}_{i_m}^{(3)}}\,\frac{\dfrac{1}{h_{i_m} - h_{i_m-1}}\,\mathbf{V}^{T}\,\mathbf{s}_{i_m}^{(3)}}{\dfrac{1}{h_{n_s}}\,\mathbf{V}^{T}\,\mathbf{s}^{(g)}} = \frac{\dfrac{1}{\beta_{i_m}}\,\dfrac{1}{h_{i_m} - h_{i_m-1}}\,\mathbf{V}^{T}\,\mathbf{s}_{i_m}^{(3)}}{\nu^{(g)}} = \frac{\dfrac{1}{\beta_{i_m}}\sum_{k=i_m}^{n_s} V_k}{\dfrac{1}{h_{n_s}}\sum_{k=1}^{n_s} V_k\,h_k} \tag{45}$$

The parameter $\Delta_{i_m}^{(3)}$ represents the ratio between the slope of the equilibrium curve of the i_mth mechanism of type-3 and the one of the global mechanism.

By means of the above parameters the n_s design conditions to be satisfied in order to avoid collapse mechanisms of type-3 can be written as

$$\mathbf{M}_{c1}^{T}\,\mathbf{I} + \mu^{(g)} - \tau^{(g)} - \nu^{(g)}\,\delta_u \leqslant \frac{1}{\beta_{i_m}}\,2\mathbf{M}_{c,i_m}^{T}\,\mathbf{I} - \delta_u\,\Delta_{i_m}^{(3)}\,\nu^{(g)} \tag{46}$$

Finally, denoting with $\rho_{i_m}^{(3)}$ the values of the ratios (34) corresponding to the fulfilment of the n_s design conditions to avoid collapse mechanisms of type-3, the following relation is obtained:

$$\rho_{i_m}^{(3)} \geqslant \frac{\sum_{i=1}^{n_c} M_{c,i1} + \mu^{(g)} - \tau^{(g)} + \nu^{(g)}(\Delta_{i_m}^{(3)} - 1)\,\delta_u}{\dfrac{2}{\beta_{i_m}}\sum_{i=1}^{n_c} M_{c,i1}} \tag{47}$$

which has to be applied for $i_m = 1, 2, 3, \ldots, n_s$.

Technological conditions

According to the above formulations, $3n_s$ design conditions have been derived directly from the extension of the upper bound theorem to the mechanism equilibrium curves. In particular, for each storey there are three design conditions to be satisfied, because three collapse mechanism typologies have been considered. As these design conditions have to be contemporaneously satisfied for each storey, the ratios (34) between the sum of the reduced plastic moments of columns of the i_mth storey and the same sum corresponding to the first-storey columns allow to satisfy the above design conditions if the following relationship is verified:

$$\rho_{i_m} = \max\left\{\rho_{i_m}^{(1)}, \quad \rho_{i_m}^{(2)}, \quad \rho_{i_m}^{(3)}\right\} \tag{48}$$

As the section of columns can only decrease along the height of the frame, the values of ρ_{i_m} (with $i_m = 1, 2, \ldots, n_s$) obtained by means of the conditions derived through the application of the upper bound theorem have to be modified in order to satisfy the following technological limitation:

$$\rho_1 \geqslant \rho_2 \geqslant \rho_3 \geqslant \cdots \geqslant \rho_{n_s} \tag{49}$$

Evaluation of the axial load in the columns at the collapse state

If the sum of the reduced plastic moments of columns of the first storey is specified, then the previously explained design conditions allow the definition, through the ratios ρ_k ($k = 1, 2, \ldots, n_s$), of the same sum corresponding to the kth storey, which guarantees that failure does not occur according to mechanisms belonging to the three examined typologies. In order to define the plastic section modulus of the columns, the evaluation of the axial load in the columns at the collapse state is required.

The evaluation of the column axial forces can be performed taking into account that, at the collapse state, the shear forces transmitted by the beams are given by

$$S = \frac{qL}{2} \pm \frac{2M_b}{L} \tag{50}$$

provided that the limit value of the uniform vertical load, given by equation (2), is not exceeded and by

$$S = \frac{qL}{2} \pm \frac{M_0 + M_b}{L} \tag{51}$$

where M_0 is provided by equation (4), in the opposite case.

Both in equation (50) and in equation (51), for positive horizontal forces (from left towards right), the sign plus is referred to the right end of the beam and the sign minus is referred to the left end of the beam.

The sum of these shear forces transmitted by the beams at each storey, above the considered one, provides the axial forces in the columns of the considered storey.

DESIGN ALGORITHM

It has been pointed out that the upper bound theorem allows the assessment of a condition for avoiding each undesired collapse mechanism, by considering the ratio between the sum of the reduced plastic moments of the kth-storey columns and the same sum corresponding to the first storey. As three different collapse mechanism typologies have been considered, there are $3n_s$ design conditions to be satisfied, which are provided by relations (35), (42) and (47). These design conditions have to be integrated by the technological condition (49). The above-mentioned relations can be used to design frames failing in global mode and, therefore, having a mechanism equilibrium curve given by equation (10), with the kinematically admissible multiplier of horizontal forces given by equation (17) and the slope given by relation (16), with $\mathbf{s} = \mathbf{s}^{(g)}$ and $H_0 = h_{n_s}$. The algorithm to solve this problem is now presented. The following steps have to be performed:

(a) Selection of the maximum displacement up to which it is desired to assure that the collapse mechanism cannot be different from the global one (i.e. up to which the mechanism equilibrium curve of the global

mechanism has to lie below those of all other mechanisms); this displacement has to be the ultimate displacement and therefore it can be evaluated as $\delta_u = \theta_{p_u} h_{n_s}$, where θ_{p_u} is the ultimate value of the plastic rotation of beams or beam-to-column connections.

(b) Computation of the storey function ξ_{i_m}, λ_{i_m}, $\zeta^{(1)}_{i_m}$, θ_{i_m}, γ_{i_m}, $\zeta^{(2)}_{i_m}$ and β_{i_m}, which are provided by equations (29), (30), (31), (37), (38), (39) and (44), respectively.

(c) Computation of the parameters $\Delta^{(t)}_{i_m}$, related to the influence of second-order effects, given by equations (32), (40) and (45).

(d) Computation of the slopes $\gamma^{(t)}_{i_m}$ of the equilibrium curves of the considered mechanisms, provided by equation (16) with $\mathbf{s} = \mathbf{s}^{(1)}_{i_m}$ and $H_0 = h_{i_m}$ for mechanism type-1, $\mathbf{s} = \mathbf{s}^{(2)}_{i_m}$ and $H_0 = h_{n_s} - h_{i_m - 1}$ for mechanism of type-2, and, finally, with $\mathbf{s} = \mathbf{s}^{(3)}_{i_m}$ and $H_0 = h_{i_m} - h_{i_m - 1}$ for type-3 mechanisms.

(e) Computation, through equations (17) and (16) (with $\mathbf{s} = \mathbf{s}^{(g)}$ and $H_0 = h_{n_s}$) of a tentative value α_t of the multiplier corresponding to the global mechanism and to the selected ultimate displacement ($\alpha_t = \alpha^{(g)} - \gamma^{(g)} \delta_u$) by imposing that the reduced plastic moment of the first storey columns is not less than the plastic moment of the beams.

(f) Computation of the limit values $\rho^{(1)}_{i_m}$, $\rho^{(2)}_{i_m}$ and $\rho^{(3)}_{i_m}$ provided by equations (35), (42) and (47), respectively.

(g) Computation, by means of equation (48), of the values of ρ_{i_m} which prevent the failure modes corresponding to the three examined collapse mechanism typologies.

(h) Modification of the computed values of ρ_{i_m} in order to satisfy the technological condition (49).

(i) Computation of the corresponding kinematically admissible multipliers $\alpha^{(1)}_{i_m}$, $\alpha^{(2)}_{i_m}$ and $\alpha^{(3)}_{i_m}$ provided by equation (19), (21) and (23), respectively.

(j) Computation of the ultimate multiplier as the minimum among all the kinematically admissible multipliers, but including the influence of second-order effects for the selected ultimate displacement:
$\alpha_u = \min\{\alpha^{(t)}_{i_m} - \gamma^{(t)}_{i_m} \delta_u;\ \text{with } i_m = 1, 2, 3, \ldots, n_s \text{ and } t = 1, 2, 3\}$ (52)

(k) If the condition:

$$|\alpha_u - \alpha_t| > \text{tolerance} \tag{53}$$

is verified, then the value of $\sum_{i=1}^{n_c} M_{c,i1}$ corresponding to $\alpha_u = \alpha^{(g)} - \gamma^{(g)} \delta_u$ has to be computed and the procedure has to be repeated starting from step (f). In the opposite case, convergence is achieved and the mechanism equilibrium curve is given by $\alpha_c = \alpha^{(g)} - \gamma^{(g)} \delta$; therefore the column sections can be derived according to the following steps.

(l) Computation of the axial force in the columns at the collapse state.

(m) Computation, for each storey, of the sum of the reduced plastic moments by means of equation (34). The reduced plastic moment of each column can be now obtained by assuming that each column provides the same contribution to the above-mentioned sum.

(n) Definition of the section of the ith column of the kth storey, by assuming that the point $(N_{ik}, M_{c,ik})$ belongs to the yielding surface.

The described design method has been applied to a wide series of structural schemes, obtained by varying the number of bays from 2 to 6 and the number of storeys from 2 to 8. The design results have been verified by means of static inelastic analyses including second-order effects. Structures failing in a global mode have been obtained in all the examined cases, pointing out the reliability of the design method.[26]

A worked example showing the practical application and accuracy of the proposed design procedure is given in the next section.

WORKED EXAMPLE

Structural scheme and preliminary design of beams

In order to show the practical application of the proposed design procedure, the seismic design of a six-bay five-storey moment resisting steel frame is presented in this section. The inelastic behaviour of the designed

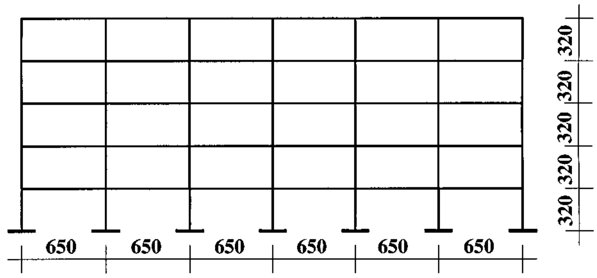

Figure 7. Structural scheme of the example frame (dimension in cm)

frame is successively examined by means of a push over static inelastic analysis confirming the fulfilment of the design objective, i.e. the location of the dissipative zones at the beam ends only with the exception of the base section of first-storey columns.

The structural scheme of the frame to be designed is shown in Figure 7. The bay span is equal to 6·50 m; the interstorey height is equal to 3·20 m. The characteristic values of the vertical loads acting on the beams are equal to 15 and 10 kN/m for permanent (G_k) and live (Q_k) actions, respectively.

The combination of actions corresponding to the frame subjected to vertical loads only is $1{\cdot}35G_k + 1{\cdot}50Q_k$. Therefore, the beams are designed to withstand a uniform vertical load equal to 35·25 kN/m. For this purpose, an IPE330 section in Fe430 steel is adopted for all the beams.

With reference to the seismic design situation, corresponding to the combination of actions $\sum G_k + \sum \psi_2 Q_k + \gamma_I A_{E_d}$ (where ψ_2 is the coefficient for the quasi-permanent value of a variable action, γ_I is the importance factor and A_{E_d} is the design value of the earthquake action), the uniform vertical load acting on the beams is ($\psi_2 = 0{\cdot}3$ for residential buildings):

$$q = 15 + 0{\cdot}3 \times 10 = 18 \text{ kN/m}$$

The plastic design resistance of beams is

$$M_{\mathrm{b,Rd}} = \frac{Zf_y}{\gamma_{M_0}} = \frac{(804 \times 10^{-6})(275 \times 10^3)}{1{\cdot}10} = 201 \text{ kN m}$$

where Z is the plastic modulus, f_y is the yield stress and γ_{M_0} is the partial safety factor. According to equation (2) the limit value of the uniform vertical load is

$$q_{\lim} = \frac{4M_{\mathrm{b,Rd}}}{L^2} = \frac{4 \times 201}{6{\cdot}50^2} \approx 19{\cdot}03 \text{ kN/m}$$

With reference to the seismic design situation, this limit value is not exceeded; therefore plastic hinges develop at the beam ends.

According to Eurocode 8,[11] the value of the period of vibration to be used for preliminary design is

$$T = 0{\cdot}085H^{3/4} = 0{\cdot}085 \times 16^{0{\cdot}75} \approx 0{\cdot}68 \text{ s} \tag{54}$$

where H is the total height of the frame.

With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor q equal to 6, the horizontal seismic forces are those given in Table I, where the value $\gamma_I = 1{\cdot}0$ has been assumed for the importance factor.

Selection of design ultimate displacement

The first step of the design procedure for failure mode control is the selection of the maximum top sway displacement up to which it is desired to assure that the collapse mechanism cannot be different from the global one.

Table I. Seismic horizontal forces

Storey	Seismic force (kN)
1	21·71
2	43·42
3	65·13
4	86·84
5	118·40

This is a very important point, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. In addition, the complete development of the collapse mechanism could be prevented by the occurrence of plastic rotation demands exceeding the local ductility supply. Therefore, a good criterion to choose the design ultimate displacement δ_u is to relate it to the plastic rotation supply of beams or beam-to-column connections by assuming $\delta_u = \theta_p H$ (where θ_p is the plastic rotation supply).

Concerning the plastic rotation supply of steel moment connections, it is traditionally measured by cyclic moment rotation tests. In the past, some researchers proposed that if a connection can reach a plastic rotation of 0·02 radian under cyclic loading, the connection can be considered sufficiently ductile to be used for seismic resistant frames.[27] Other researchers have established that, in the case of severe earthquakes, the required plastic rotation can reach more than 0·03 radian.[28–30].

This topic is particularly important, as confirmed by the recent earthquakes of Northridge (17 January 1994) and Hyogoken-Nambu (Kobe, 17 January 1995), and deserves further investigations; however, for the purpose of failure mode control, the value 0·04 rad is herein suggested aiming to assure an increased safety level against the influence of second order effects under seismic loads.

As a consequence of the above considerations, the design value of the top sway plastic displacement has been assumed to be equal to $0{\cdot}04 \times 1600 = 64$ cm.

Computation of storey functions ξ_{i_m}, λ_{i_m}, $\zeta^{(1)}_{i_m}$, θ_{i_m}, γ_{i_m}, $\zeta^{(2)}_{i_m}$ and β_{i_m}

The parameter $\tau^{(g)}$, given by equation (28), is equal to zero, because plastic hinges develop at the beam ends ($x_{jk} = 0$). For the same reason, the parameters $\zeta^{(1)}_{i_m}$ and $\zeta^{(2)}_{i_m}$ are all equal to zero.

The parameter $\mu^{(g)}$, given by equation (26), is equal to

$$\mu^{(g)} = 2 \times 6 \times 5 \times 201 = 12\,060 \text{ kN m}$$

while the overall overturning moment is given by

$$\sum_{k=1}^{n_s} F_k h_k = 3978{\cdot}56 \text{ kN m}$$

The storey functions ξ_{i_m}, λ_{i_m}, θ_{i_m}, γ_{i_m} and β_{i_m}, given by equations (29), (30), (37), (38) and (44), respectively, are presented in Table II.

Computation of the parameters $\Delta^{(t)}_{i_m}$

The total vertical load acting at each storey is given by

$$V_k = 6 \times 18 \times 6{\cdot}50 = 702 \text{ kN}$$

Therefore, the parameter $v^{(g)}$, given by equation (27), is equal to 2106 kN.

The parameters $\Delta^{(t)}_{i_m}$, related to the influence of second-order effects, are given in Table III according to equations (32), (40) and (45).

Table II. Values of storey functions

Storey i_m	ξ_{i_m}	λ_{i_m}	θ_{i_m}	γ_{i_m}	β_{i_m}
1	0	0·2698	1·00	1·0000	0·2698
2	0·20	0·5222	0·80	0·7302	0·2524
3	0·40	0·7397	0·60	0·4778	0·2175
4	0·60	0·9048	0·40	0·2603	0·1651
5	0·80	1·0000	0·20	0·0952	0·0952

Table III. Values of the parameters related to second-order effects

Storey i_m	$\Delta_{i_m}^{(1)}$	$\Delta_{i_m}^{(2)}$	$\Delta_{i_m}^{(3)}$
1	6·1775	1·0000	6·1764
2	2·8725	1·1413	5·2829
3	1·8025	1·3954	4·5985
4	1·2895	1·9208	4·0385
5	1·0000	3·5003	3·5003

Table IV. Slopes of the mechanism equilibrium curves

Storey i_m	$\gamma_{i_m}^{(1)}$	$\gamma_{i_m}^{(2)}$	$\gamma_{i_m}^{(3)}$
1	0·032694	0·005293	0·032694
2	0·015204	0·006041	0·027965
3	0·009542	0·007386	0·024342
4	0·006826	0·010168	0·021377
5	0·005293	0·018528	0·018528

Computation of the slopes of the mechanism equilibrium curves

The slopes of the mechanism equilibrium curves depend only on the magnitude and distribution of both vertical loads and horizontal forces. Therefore, as already stated, they are computed through equation (16) by properly specifying H_0 and **s**. These slopes are given in Table IV for the examined frame.

Design of column sections

The application of the design conditions to be satisfied to avoid all undesired collapse mechanisms [equations (35), (42) and (47)] requires the preliminary design of the first-storey columns. For this preliminary design, it is assumed that the reduced plastic moment of first-storey columns is not less than the beam plastic moment. According to this assumption, the sum of the reduced plastic moments of first-storey columns is initially assumed equal to

$$\sum_{i=1}^{n_c} M_{c,i1} = 7 \times 201 = 1407 \text{ kN m}$$

This value corresponds to an attempt ultimate multiplier $\alpha_t = \alpha^{(g)} - \gamma^{(g)}\,\delta_u = 3{\cdot}0461$.

The application of steps from (**f**) to (**k**) of the design algorithm leads to an ultimate multiplier equal to 3·8315, therefore a second iteration is necessary.

The new value of $\sum_{i=1}^{n_c} M_{c,i1}$, corresponding to $\alpha_u = 3{\cdot}8315$, is equal to 4531·7632 kN m. As a consequence, the application of the design conditions to avoid undesired collapse mechanisms, i.e. equations (35), (42) and

Table V. Values of the parameters $\rho_{i_m}^{(t)}$

Storey i_m	$\rho_{i_m}^{(1)}$	$\rho_{i_m}^{(2)}$	$\rho_{i_m}^{(3)}$	ρ_{i_m}
1	0·4034	1·0000	0·7017	1·0000
3	0·6706	0·5750	0·6228	0·8202
3	0·8202	0·2087	0·5145	0·8202
4	0·7937	− 0·0402	0·3768	0·7937
5	0·5322	− 0·1128	0·2097	0·5322

Table VI. Values of horizontal force multipliers

Mechanism i_m	$\alpha_{i_m}^{(1)}$	$\alpha_{i_m}^{(2)}$	$\alpha_{i_m}^{(3)}$	$\alpha_{u.i_m}^{(1)}$	$\alpha_{u.i_m}^{(2)}$	$\alpha_{u.i_m}^{(3)}$
1	8·4422	4·1703	8·4422	6·3498	3·8315	6·3498
2	5·1311	4·6008	7·4038	4·1580	4·2142	5·6141
3	4·4422	5·7623	8·5928	3·8315	5·2896	7·0349
4	4·2684	8·1311	10·9535	3·8315	7·4804	9·5843
5	4·1703	12·7323	12·7323	3·8315	11·5465	11·5465

Table VII. Design values of column bending moment (M) and axial force (N_{int} and N_{ext} for internal and external columns, respectively)

Storey	M (kN m)	N_{int} (kN)	N_{ext} (kN)
1	647·39	585·00	601·73
2	531·03	468·00	481·38
3	531·03	351·00	361·04
4	513·85	234·00	240·69
5	344·57	117·00	120·35

(47), leads to the values of $\rho_{i_m}^{(t)}$ given in Table V (step (f)). In the same table, the values of ρ_{i_m} satisfying the technological condition (49) are also presented (steps (g) and (h)).

The kinematically admissible multipliers $\alpha_{i_m}^{(t)}$ and the ultimate multipliers $\alpha_{u\,i_m}^{(t)} = \alpha_{i_m}^{(t)} - \gamma_{i_m}^{(t)}\,\delta_u$, corresponding to the design ultimate displacement, are given in Table VI. This table points out that the minimum value of $\alpha_{i_m}^{(t)} - \gamma_{i_m}^{(t)}\,\delta_u$ is equal to 3·8315. Therefore, convergence has been achieved.

It is useful to note that the minimum value of the ultimate multiplier corresponds not only to the global mechanism, but also to some other partial mechanisms. This means that the development of plastic hinges according to the global mode is assured for $\delta < \delta_u$. On the contrary, column hinging can develop for $\delta \geqslant \delta_u$ (according to mechanism type 1 for $i_m = 3, 4$ or 5). However, this last possibility has no design interest, because the condition $\delta \geqslant \delta_u$ corresponds to plastic rotation demands exceeding the local ductility supply (i.e. the suggested limit value equal to 0·04 radian). In other words, not only the failure mode, but also the local ductility control have been obtained by means of the proposed design procedure.

By means of the ratios ρ_{i_m} given in Table V the sum of the reduced plastic moments of the columns is defined at each storey. As a consequence, by assuming that any given storey all the columns contribute equally to the sum mentioned above, the reduced plastic moment of each column can be defined. The values of these bending moments are given in Table VII, where the axial forces corresponding to the complete development of the global mechanism are also presented.

The above design values of the internal actions allow to dimension the column sections by means of the M–N interaction domain which, with reference to double T sections, can be expressed through the following

Table VIII. Design values of the column plastic moduli and corresponding sections chosen from standard HEB shapes

Storey	Internal columns		External columns	
	Z_{min} (cm^3)	Section	Z_{min} (cm^3)	Section
1	2682	HEB360	2702	HEB400
2	2182	HEB340	2182	HEB340
3	2142	HEB320	2142	HEB320
4	2062	HEB320	2062	HEB320
5	1362	HEB280	1362	HEB280

relation:

$$\frac{N}{Af_y/\gamma_{M_0}} + 0{\cdot}9\,\frac{M}{Zf_y/\gamma_{M_0}} = 1 \tag{55}$$

for $N > 0{\cdot}10\ Af_y/\gamma_{M_0}$ and

$$\frac{M}{Zf_y/\gamma_{M_0}} = 1 \tag{56}$$

in the opposite case.

With reference to standard HE shapes, the geometrical properties of the section lead to a relation between the area A and the plastic modulus Z of the section which can be expressed in the following form:

$$A = aZ^b \tag{57}$$

For the sake of simplicity, this relationship can be combined with the M–N interaction domain in order to compute the minimum values that the plastic moduli of columns must have to avoid undesired collapse mechanisms.

With reference to HEB shapes ($a = 1{\cdot}8214$ and $b = 0{\cdot}5798$) and Fe430 steel, the required minimum values of the column plastic moduli are given in Table VIII with the corresponding sections chosen from the standard shapes. As a consequence, the structural weight of the designed frame is equal to 23·82 tons.

Push over static inelastic analysis

As the column sections have been chosen from the standard shapes, according to Table VIII, they obviously have some overstrength with respect to the minimum required plastic flexural resistance. As a consequence, a slight increase (4·2527 against 4·1703) of the kinematically admissible multiplier of horizontal forces, corresponding to the global mechanism is obtained. Therefore, the global mechanism equilibrium curve of the designed frame is

$$\alpha = 4{\cdot}2527 - 0{\cdot}005293\delta$$

The overall behaviour of the designed frame can be well represented by means of the bilinear curve OAB (Figure 8). The point A is obtained as the intersection between the global mechanism equilibrium curve and the straight line representing the linear elastic behaviour. In the same figure, the horizontal force multiplier versus top sway displacement curve obtained through a push over static analysis, including both geometrical and mechanical non-linearities, is also presented. This analysis has been performed under displacement control aiming at the evaluation of the softening branch of the above behavioural curve. The comparison between this softening branch and the global mechanism equilibrium curve provides a first confirmation of the accuracy of the proposed design procedure.

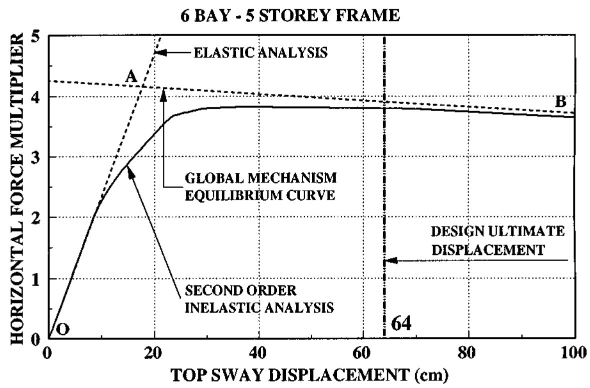

Figure 8. Behavioural curve of the designed frame and comparison with the corresponding bilinear approximation

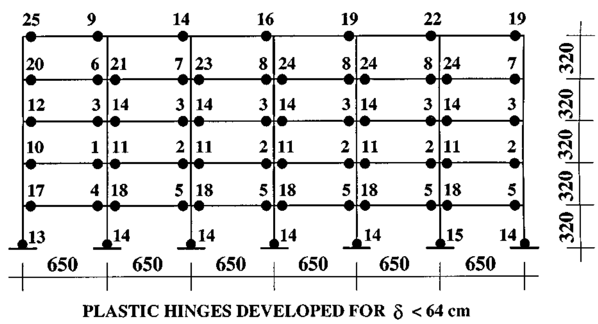

Figure 9. Pattern of yielding of the designed frame

The second and most important confirmation of the fulfilment of the design objective is represented by the plastic hinges developed before the occurrence of the design ultimate displacement. In fact, the pattern of yielding is shown in Figure 9 through the plastic hinge formation sequence obtained from the push over static inelastic analysis. Column yielding occurs only at the base of first storey columns confirming the reliability of the proposed design procedure. The complete development of the collapse mechanism does not occur and the structure is still stable even when the design ultimate displacement has been attained.

A great number of moment resisting frames have been designed by means of the proposed procedure.[26] The number of bays varied from 1 to 6, the number of storeys from 2 to 8. The push over static inelastic analyses confirmed in all cases the fulfilment of the design objective, i.e. the development of a pattern of yielding according to the global mechanism.

Comparison with the member hierarchy criterion

In order to point out the advantages of the proposed design procedure, it is useful to compare the inelastic performances of the designed frame with those obtained by means of more simple design criteria. In particular, two empirical approaches are considered.

The first approach is the simple member hierarchy criterion requiring that at any beam-to-column joint the sum of the column plastic moments has to be greater than the sum of the plastic moments of the connected beams. Exception is made for the roof level. The basic assumption of this simple criterion is that, after the formation of plastic hinges in beams, the points of contraflexure are located generally close to the mid-height of columns.

Inspite of this simple design criterion suggested in modern seismic codes,[11–14] it does not lead to the development of a collapse mechanism of global type.[15,17,18] In particular, by means of inelastic analyses, it

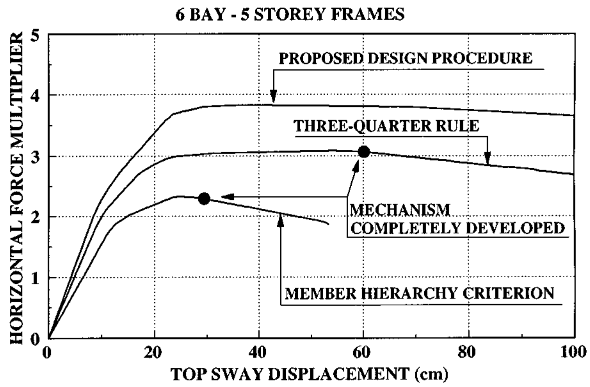

Figure 10. Comparison between the behavioural curve of the frame designed according to the proposed procedure and those of the frames designed, respectively, according to the simple member hierarchy criterion and according to the three-quarter rule

has been recognized that the distribution of bending moments in columns is far from the assumption that the points of contraflexure are located at the column midheight.[17,18,31] For this reason, a second approach aiming to the strong-column weak-beam design concept has been recently proposed by Han-Seon Lee.[18] The revised rule is based on the observation that, at any joint, three quarters of the sum of beam plastic moments is taken by the lower column plastic moment and the remaining quarter is taken by the upper column. According to this distribution of column bending moments in inelastic range, the following three-quarter rule for strong-column and weak-beam design has been suggested:

$$M_c \geqslant 0{\cdot}75\, \eta \sum M_b \tag{58}$$

where M_c is the column plastic moment reduced accounting for the influence of the axial force, M_b is the beam plastic moment and η is a factor taking into account beam strain-hardening ($\eta \approx 1{\cdot}10$).

As a result of the application of the first design approach, i.e. the member hierarchy criterion in its classical form ($\sum M_c \geqslant \sum M_b$ at any joint), the frame members are IPE330, HEB240 and HEB200 for beams, internal columns and the external columns, respectively. Conversely, the fulfilment of the three-quarter rule according to equation (58) requires, for the given beam section (IPE330), the adoption of HEB300 and HEB240 for internal and external columns, respectively.

With reference to the frames designed above and to the combinations of actions already defined in the previous section, the internal actions have been computed by means of elastic analyses, static and dynamic. In addition, geometrical imperfections have been accounted according to Eurocode 3 and second-order effects have been evaluated by means of the moment amplification method. Finally, resistance and stability checks of members have been performed according to Eurocode 3.

The frame designed according to the simple member hierarchy criterion weighs 18·15 tons while that designed according to the three-quarter rule weighs 21·57 tons. Therefore, both these empirical design rules lead to a structural solution whose weight is less than that (23.82 tons) required for assuring a collapse mechanism of global type. Therefore, for the examined structural scheme, the design procedure leading to the development of the global mechanism requires an increase of the structural weight of about 30 per cent with respect to the simple member hierarchy criterion and about 10 per cent with respect to the three-quarter rule.

The comparative examination of the inelastic behaviour of the designed frames is presented in Figure 10 through the corresponding curves relating the multiplier of seismic horizontal forces to the top sway displacement, as obtained from push over static inelastic analyses.

It is evident that the frame designed according to the proposed design procedure is characterized, with respect to those designed according to empirical rules for failure mode control, by a significant increase of strength, plastic redistribution capacity and ductility. In addition, in Figures 11 and 12 the collapse mechanisms of the frames designed according to the above empirical rules are represented. These figures

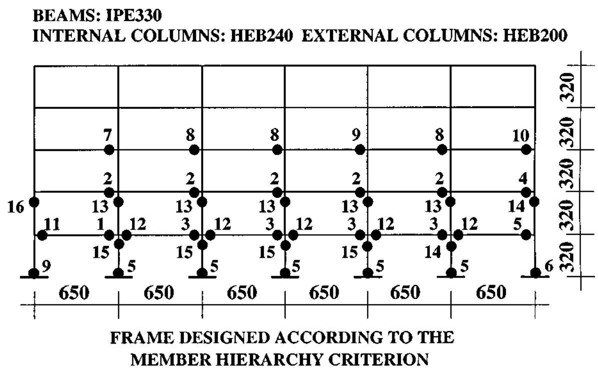

Figure 11. Pattern of yielding of the frame designed according to the simple member hierarchy criterion

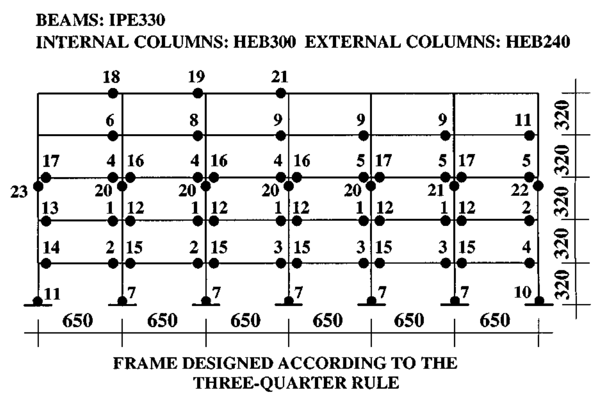

Figure 12. Pattern of yielding of the frame designed according to the three-quarter rule

demonstrate that both the use of the simple member hierarchy criterion and the use of the three-quarter rule are able to avoid the development of a soft storey mechanism, but they do not prevent column hinging due to the occurrence of partial mechanisms.

Finally, the seismic inelastic performances of the designed frames have been compared further by computing the q-factor through an energy approach based on push over static inelastic analysis,[21,32] leading to the following relation:

$$q = \left(\frac{W_u}{W_1} \right)^{1/2} \tag{59}$$

where W_u is the work done by horizontal forces up to the attainment of the ultimate displacement and W_1 is the elastic work under the design horizontal forces ($\alpha = 1$).

The q-factor of the frame dimensioned according to the proposed design procedure is about two times that of the frame designed according to the simple member hierarchy criterion (8·87 against 4·77) and about 25 per cent greater than that of the frame designed according to the three-quarter rule (8·87 against 7·18). These results are based on the assumption that the plastic rotation capacity of members, i.e. the ratio between ultimate plastic rotation and first yielding rotation, is equal to 4.

The above results are particularly important, because they underline that a relatively small increase of the structural weight, representing the price of the global mechanism, leads to an excellent improvement of the seismic inelastic behaviour.

A comparative analysis of the performances of seismic resistant steel frames designed according to different criteria confirms the excellent behaviour of the frames designed according to the proposed design procedure based on the theoretical analysis of the mechanism equilibrium curves.[33]

CONCLUSIONS

A new method for designing moment resisting steel frames failing in a global mode has been presented in this paper. The method is based on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. This allows to include into the design process the influence of second-order effects, which plays a very important role in the seismic design of steel frames.

The reliability of the proposed design procedure has been also demonstrated through its application to a six-bay–five-storey frame, leading to the fulfilment of the design objective, i.e. the development of a collapse mechanism of global type, as it has been confirmed by the results of a push over static inelastic analysis.

Finally, the results of the proposed design procedure have been compared with those obtained by using empirical approaches for failure mode control, such as the member hierarchy criterion and the three-quarter rule. This comparison has underlined as only the proposed theoretical approach is able to assure the development of a pattern of yielding according to the global failure mode.

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